Math Contest Answers

1. At 7:00 AM A member of the Board of Elections observes 8 people waiting at the entrance to the poll. In how many different orders can the voters be allowed to enter?

Since order is important we have 8P8 = 8! or 40320 orderings

2. Among the 8 people who arrived at 7:00 are two siblings Sam and Pat. What's the probability that they are separated in line by exactly 2 people (to the nearest thousandth)?

Possible arrangements are:

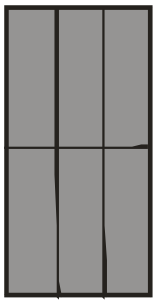
S _ _ P _ _ _ _, _ S _ _ P _ _ _ , _ _ S _ _ P _ _,
_ _ _ S _ _ P _, _ _ _ _ S _ _ P

or 5 arrangements with Sam first plus 5 more with Pat first. The remaining 6 people may be arranged in 6! = 720 ways. This makes 10 x720 = 7200 orderings and gives a probability of

7200/40320 ~ 0.179

3. The curtain to the voting machine is divided by pleats into identical rectangles. If the perimeter of each rectangle is 8 feet, What are the dimensions of the whole curtain?

Note that the top and bottom halves of the curtain are squares. Each narrow rectangle has a length that is 3 times its width (x). The perimeter = 2x + 2(3x) = 8x = 8. Solving for x we get x = 1 foot. The dimensions of each narrow rectangle is 1 foot by 3 feet. The whole curtain is 3 feet by 6 feet.

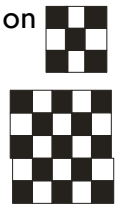


4. Each member of the Board of Elections is supposed to serve a 3 hour shift. A member started his shift at 7:00 AM. and his replacement is late. In fact his replacement doesn't arrive until the hands on the clock overlap. How late is the replacement (to the nearest minute)?

The shift was supposed to end at 10:00 AM. The minute hand moves 360/60 = 6 degrees per minute. The hour hand moves 30/60 = 0.5 degrees per minute. 10:00 is 300 degrees from the beginning (12). We want to solve:

$300+0.5x=6x \quad \rightarrow \quad x=54.\overline{54}\approx 55$ minutes

5. The floor of the polling place is tiled in a checkerboard pattern with alternating black and white tiles. There are black tiles in each of the 4 corners of the room and the diagonals of the room total to 41 tiles. If the tiles are 12 inch squares, find the dimensions of the room.



A 3 by 3 square has 2(3) - 1 = 5 total black tiles on the diagonals. A 5 by 5 square has 2(5) - 1 = 9 total tiles on the diagonals. Therefore our n by n room has $2n - 1 = 41 \rightarrow n = 21$ rows and columns. Hence the room is 21 feet by 21 feet (252 by 252 inches is also acceptable).

6. A reporter conducting exit polls tries to guess presidential candidate chosen by voters he interviews. He guesses wrong on the first voter but gets the next three right. What is the smallest number of consecutive guesses he must get right to be at least 90% correct with his guesses?

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If he guesses correctly x consecutive times we want:

$$\frac{3 + x}{4 + x} \geq \frac{9}{10}$$

or $30 + 10x \geq 36 + 9x$ ® $x \geq 6$. $3 + 6 = 9$ and $4 + 6 = 10$, so 6 is the minimum number of times he must guess correctly.

7. A resident of Eastwick arrives at the poll to find they have a choice among 3 presidential tickets, 1 of two candidates for US Senate, 1 of two candidates for RI Senate, 1 of 3 candidates for RI house, 3 out of 5 members for town council and 3 out of 8 candidates for the local school board. How many different ways can the voter fill in his ballot?

Since nothing in the problem indicates the ordering of the town council members or board of education members is important these part of the problems are combination calculations. Using the Fundamental Theorem of Counting

$3 \times 2 \times 2 \times 3 \times {}_5C_3 \times {}_8C_3 = 36 \times 10 \times 56 = 20160$

possible ballot orderings.

8. A collection of coins is made up of an equal number of pennies, nickels, dimes and quarters. What is the largest possible value for a collection that is less than $5.00?

.25 + .10 + .05 +.01 = .41

5.00/.41 = 12.195… The integer part being 12.

12(.41) = $4.92

9. The Eastwick *Sentinel* advertises that its Sunday paper costs 1/3 of the price of the rest of the week's daily papers. If a weekly subscription costs between $5.95 and $6.05, what's the cost of a Sunday paper plus a daily paper?

We want

$$5.95 \leq 6x + {}^1/_3(6x) \leq 6.05 \quad \text{or}$$
$$5.95 \leq 8x \leq 6.05$$
$$0.74375 < x < 0.75625$$

The daily papers have to sell for a integer number of cents; thus $0.75 is the cost for a daily paper and $1.50 is the cost for a Sunday paper and the sum is $2.25. Note that at $0.74 for a daily paper the weekly total would be $5.94 and at $0.76 the weekly total is $6.08. Both are out of the stated range.

10. How many four-digit whole numbers occur such that the left-most digit is odd, the second digit is even, and all four digits are different?

First digit has 5 choices as does the second, the 3rd has 10 - 2 = 8 choices and the final digit has 8 - 1 = 7 We have $5 \times 5 \times 8 \times 7 = 1400$ four digit numbers.

11. Given the length of each side of the square ABCD is 1, find the shaded area of the crescent shaped loon ABC. Note: the shape is bounded by semicircle ABC and the arc of a circle centered at point D.

Area of segment formed by circle centered a D and diagonal line AC is ${}^{R^2}/_2 x(\theta - \sin\theta) = 0.5(\pi/2 - \sin\pi/2) = \pi/4 - 0.5$. Area of semicircle ABC = $\pi({}^{\sqrt{2}}/_2)^2/_2 = \pi/4$. Area of the loon is area of the semicircle minus area of the segment $(\pi/4 - 0.5) - \pi/4 = 0.5$